ANALYTICAL SOLUTION OF THE HEAT-CONDUCTION PROBLEM IN A BOUNDED MULTILAYER DOMAIN WITH LOCAL VOLUME SOURCES IN THE LAYER

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The three-dimensional heat-conduction problem is solved for a multilayer domain with a local source in one of the layers. The numerical results of the solution are analyzed in application to the layered structure of a semiconductor integrated circuit.

In engineering we encounter structures in the form of multilayer configurations with local volume sources of heat release in one of the layers.

Typical of such structures is a semiconductor integrated circuit (IC) consisting of successive layers of silicon, silicon dioxide, silicon nitride, aluminum, etc. Other layer configurations are possible as well. Inasmuch as the performance characteristics and parameters of the elements of such systems often depend strongly on their temperature, it becomes necessary to solve the heat-conduction problem in this kind of multilayer domain. The need to solve the problem is further stipulated by the fact that currently existing thermometric methods, including infrared (IR) radiometry and heat sensors, provide information only about the surface temperature averaged over the area of geometric resolution of the corresponding method. Also, the dimensions of these areas are often greater than the dimensions of the heat sources, so that the temperature values afforded by measurements are distinctly too low.

In the present paper we solve the steady heat-conduction problem for a multilayer domain, making it possible to estimate the influence of the covering layers on the temperature of an internal source and its deviation from the value measured in an outer surface layer.

The investigated domain and adopted coordinate system are shown in Fig. 1.

The system of equations describing the temperature field in the given domain has the form

$$\Delta\theta_r = -\psi\{e[x-(\varepsilon-l_1)] - e[x-(\varepsilon+l_1)]\}$$

$$\times\{e[y-(\eta-l_2)] - e[y-(\eta+l_2)]\}\; e(h-z)\,\nu(r),$$

$$r=0,\ 1,\ 2,\ \ldots,\ k,\quad \nu(r)=\begin{Bmatrix} 1 \text{ for } r=0, \\ 0 \text{ for } r=1,\ 2,\ 3,\ \ldots,\ k \end{Bmatrix},$$

$$\frac{\partial\theta_k}{\partial z_k}=0 \quad \text{for} \quad z=z_k, \tag{1}$$

TABLE 1. Temperature Distributions (°C) over Layer Boundaries

Structure	Layer	Coordinate x, mm, for y = η = ε = 1.5								
		0,3	0,6	0,9	1,2	1,3	1,35	1,40	1,45	1,50
$Si—Al—SiO_2$	SiO_2	41,2	42,2	44,3	48,9	52,6	56,0	62,8	83.8	194,8
	Al	41,2	42,2	44,3	48,9	52,6	56,0	62,8	83,7	196,6
	Si	41,2	42,2	44,3	48,9	52.6	56,0	62,8	83,7	196,6
$Si—SiO_2—Al$	Al	41,2	42,2	44,3	48,9	52,7	56,3	63,7	85.6	139
	SiO_2	41,2	42,2	44,3	48,9	52,7	56,3	63,7	85,6	139,2
	Si	41,2	42,2	44,3	48,9	52,7	56,0	62,6	83,2	208,3
Si—Al	Al	41,2	42,2	44,3	48,9	52,6	56,0	62,8	83,8	197
	Si	41,2	42,2	44,3	48,9	52,6	56,0	62,8	83,8	197,3
$Si—SiO_2$	SiO_2	41,2	42,2	44,3	48,9	52,7	56,1	62,9	84.2	216,4
	Si	41,2	42,2	44,3	48,9	52,7	56,1	62,8	84,3	218,6
Si	Si	41,2	42,2	44,3	49	52,7	56,1	62,8	84,4	219,2

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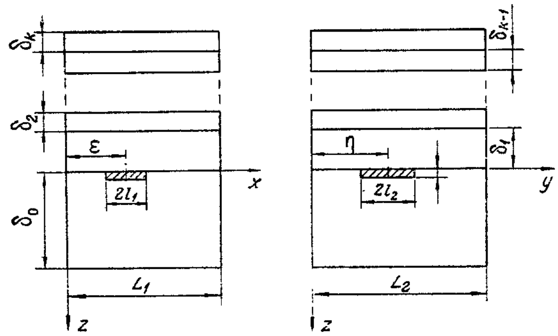

Fig. 1. Multilayer domain (the height of the hatched zone is h).

$$\frac{\partial\theta_0}{\partial z_k} = -\frac{\mathrm{Bi}}{L_1}\theta_0 \text{ for } z=\delta_0,$$

$$\theta_{i+1}=\theta_i,\quad \lambda_{i+1}^0\frac{\partial\theta_{i+1}}{\partial z}=\lambda_i^0\frac{\partial\theta_i}{\partial z} \quad \text{for } z=z_i,$$

$$z_i=-\sum_{j=1}^{i}\delta_j,\quad i=0,\ 1,\ 2,\ \ldots,\ k,\quad z_0=0.$$

We subject the system (1) to a finite integral transformation with kernel

$$K_{mn}(x,\ y)=A_mB_n\cos\lambda_m x\cos\lambda_n y,$$

where $A_0=1\sqrt{L_1}$, $A_m=\sqrt{2/L_1}$, $B_0=1\sqrt{L_2}$, $B_n=\sqrt{2/L_2}$, $\lambda_m=m\pi/L_1$, $\lambda_n=n\pi/L_2$, $m,\ n=0,\ 1,\ 2,\ \ldots,\ \infty$.

The transform of system (1) has the form

$$\frac{d^2\theta_{rmn}}{dz^2}-\lambda_{mn}^2\theta_{rmn}=-\psi_{mn}e(h-z)\nu(r),$$

$$d\theta_{kmn}/dz=0 \quad \text{for} \quad z=z_k,$$

$$\frac{d\theta_{0mn}}{dz}=-\frac{\mathrm{Bi}}{L_1}\theta_{0mn} \quad \text{for} \quad z=\delta_0, \tag{2}$$

$$\theta_{(i+1)mn}=\theta_{imn},\quad \lambda_{i+1}^0\frac{d\theta_{(i+1)mn}}{dz}=\lambda_i^0\frac{d\theta_{imn}}{dz} \quad \text{for} \quad z=z_i,$$

where

$$\theta_{rmn}(z)=\int_0^{L_1}\int_0^{L_2}\theta_r(x,\ y,\ z)K_{mn}(x,\ y)\,dxdy,$$

$$\psi_{mn}=\frac{4\psi A_mB_n}{\lambda_m\lambda_n}\cos\lambda_m\varepsilon\sin\lambda_m l_1\cos\lambda_n\eta\sin\lambda_n l_2,$$

$$\psi_{m0}=\frac{4\psi A_mB_0l_2}{\lambda_m}\cos\lambda_m\varepsilon\sin\lambda_m l_1,$$

$$\psi_{0n}=\frac{4\psi A_0B_nl_1}{\lambda_n}\cos\lambda_n\eta\sin\lambda_n l_2,$$

$$\psi_{00}=4\psi A_0B_0l_1l_2,\quad \lambda_{mn}=\sqrt{\lambda_m^2+\lambda_n^2}.$$

The solution of system (2) at $z = z_r$ is

$$\theta_{rmn}(z_r)=\frac{\psi_{mn}}{\lambda_{mn}^2}R(m,\ n)\alpha_r(m,\ n),$$

$$\theta_{r00}(z_r)=\psi_{00}h\left(\delta_0+\frac{L_1}{\mathrm{Bi}}-\frac{h}{2}\right), \tag{3}$$

where

$$R(m;\ n)=\frac{1-\mathrm{ch}\,\lambda_{mn}h+G(m,\ n)\,\mathrm{sh}\,\lambda_{mn}h}{\sigma_k+\frac{G(m,\ n)}{\lambda_0^0}\gamma_k},$$

$$\alpha_r(m,\ n)=\frac{\sigma_{k-r}}{\prod\limits_{i=0}^{r}\mathrm{ch}\,\lambda_{mn}\delta_i},\quad G(m,\ n)=\frac{\lambda_{mn}+\frac{\mathrm{Bi}}{L_1}T}{\lambda_{mn}T+\frac{\mathrm{Bi}}{L_1}},$$

$$T=\mathrm{th}\,\lambda_{mn}\delta_0,\ T_i=\mathrm{th}\,\lambda_{mn}\delta_i,\ i=1,\ 2,\ 3,\ \ldots,\ k,$$

$$\sigma_0=\sigma_1=1,\ \sigma_i=\sigma_{i-1}+\frac{T_{k+1-i}}{\lambda_{k+1}^0}\sum_{j=2}^{i}\lambda_{k+2-j}^0T_{k+2-j}\sigma_{j-2},$$

$$\gamma_0=0,\ \gamma_1=\lambda_k^0T_k,\ \gamma_i=\gamma_{i+1}+\lambda_{k+1-i}^0T_{k+1-i}\left(1+\sum_{j=2}^{i}\frac{T_{k+2-j}}{\lambda_{k+2-j}^0}\gamma_{j-2}\right)$$

$$\text{for}\quad i=2,\ 3,\ 4,\ \ldots,\ k.$$

The inverse transform of (3) takes the form

$$\theta_r(x,\ y,\ z_r)=A_0B_0\psi_{00}h\left(\delta_0+\frac{L_1}{\mathrm{Bi}}-\frac{h}{2}\right)+\sum_{m=1}^{\infty}\frac{A_mB_0\psi_{m0}}{\lambda_m^2}R(m,\ 0)$$
$$\times\alpha_r(m,\ 0)\cos\lambda_mx+\sum_{n=1}^{\infty}\frac{A_0B_n\psi_{0n}}{\lambda_n^2}R(0,\ n)\,\alpha_r(0,\ n)\cos\lambda_ny$$
$$+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{A_mB_n\psi_{mn}}{\lambda_{mn}^2}R(m,\ n)\,\alpha_r(m,\ n)\cos\lambda_mx\cos\lambda_ny. \quad (4)$$

As an example for numerical computation, we consider a three-layer structure ($k=2$) corresponding to a semiconductor IC of the MOS (metal — oxide — semiconductor) transistor type, viz., silicon – silicon dioxide – aluminum ($Si-SiO_2-Al$).

A volume source is situated in the silicon layer at the $Si-SiO_2$ interface. To estimate the influence of the layers of the structure on the source temperature we carry out a parallel investigation of other layer configurations, including single- and double-layer structures. We obtain the estimate for dimensions consistent with a real IC.

The thermophysical properties used below for SiO_2 and Al are taken from [1], and those for silicon from [2].

The input data for the numerical computation are

$$L_1=L_2=3\cdot10^{-3}\,[\mathrm{m}],\ l_1=l_2=2\cdot10^{-5}\,[\mathrm{m}],\ h=1\cdot10^{-6}\,[\mathrm{m}],\ \varepsilon=\eta=L_1/2,$$
$$\lambda_0^0=80\,[\mathrm{W/m\cdot{}^\circ K}],\ \delta_0=3\cdot10^{-4}\,[\mathrm{m}],\ \lambda_1^0=2\,[\mathrm{W/m\cdot{}^\circ K}],\ \delta_1=3\cdot10^{-6}\,[\mathrm{m}],$$
$$\lambda_2^0=210\,[\mathrm{W/m\cdot{}^\circ K}],\ \delta_2=1.2\cdot10^{-6}\,[\mathrm{m}],\ P=1\,[\mathrm{W}],$$

where λ_0^0 and δ_0 are the thermal conductivity and thickness of the Si layer, λ_1^0 and δ_1 are the same for the SiO_2 layer, and λ_2^0 and δ_2 are the same for the Al layer.

The results of the computations for $\mathrm{Bi}=0.1$, executed on a BÉSM-6 digital computer, are summarized in Table 1. The temperature increases are calculated for coordinates $0<x\leq\varepsilon$ and $y=\eta$ at the boundaries of one-, two-, and three-layer structures for various layer configurations.

It is evident from the table that the SiO_2 layer has scarcely any detectable influence on the temperature of the source in a two-layer structure. In the three-layer structure $Si-SiO_2-Al$ the sound temperature is somewhat lower (about 5%) than in the absence of covering layers. The temperatures of the faces of SiO_2 and Al in this structure practically coincide, but they are much lower (about 36%) than the source temperature for the case of a single-layer Si structure. Thus, experimental measurements of the source temperature from a top layer of aluminum will yield diminished results.

In the structures $Si-Al$ and $Si-Al-SiO_2$ the source temperature scarcely differs from the temperatures of the layers above the source; i.e., with good geometric resolution of the experimental method measurements over the surface above the source will yield sufficiently precise information. The source temperature in this case is ≈10% lower than for the single-layer Si structure.

The foregoing results confirm the legitimacy of simplified calculations neglecting the influence of the layers and treating the heat-conduction problem in the crystal of a semiconductor IC as in a homogeneous Si domain. Experiments on the source temperature from the surface of an IC having an $Si-SiO_2-Al$ structure yield excessively low results.

Analogous calculations of the temperatures on the faces of the structure $Si-SiO_2-Al$ for $Bi=0.75\cdot10^{-3}$ show that the external heat-transfer rate has virtually no effect on the relief of the temperature field.

NOTATION

$\Delta=\partial^2/\partial x^2+\partial^2/\partial y^2+\partial^2/\partial z^2$, Laplace operator; $\theta_r(x, y, z)=T_r(x, y, z)-T_{me}$; $T_r(x, y, z)$, temperature in the r-th layer; T_{me}, temperature of medium; λ_i^0, δ_i, thermal conductivity and thickness of i-th layer; $\psi=P/\lambda_0^0V$; P, power of local source; $V=2l_1\times2l_2\times h$; e(x), unit Heaviside function; α, heat-transfer coefficient; ε,η, center coordinates of source; k number of layers covering the source.

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SOLUTION OF THE UNSTEADY HEAT-CONDUCTION EQUATION IN AN INHOMOGENEOUS MEDIUM

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The solution of an unsteady two-dimensional heat-conduction problem in an inhomogeneous medium is investigated by using differential operators.

If there are no heat sources or sinks within a body, the unsteady two-dimensional heat-conduction problem is described by the equation

$$c\gamma\frac{\partial T}{\partial t}=\lambda\left(\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}\right)+\frac{\partial\lambda}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial\lambda}{\partial y}\frac{\partial T}{\partial y},\qquad(1)$$

where the thermal conductivity $\lambda=\lambda(x, y)$, the density $\gamma=\gamma(x, y)$, and the specific heat $c=c(x, y)$ are given functions of the coordinates x and y.

We seek the solution of Eq. (1) which satisfies appropriate boundary conditions [1] and has the form

$$T=\tau(t)\Psi(x, y).\qquad(2)$$

Substituting (2) into (1) and introducing the separation of variables parameter $-\nu^2$, we obtain the two equations

$$\frac{d\tau}{dt}=-\nu^2\tau;\qquad(3)$$

$$\Delta\Psi+\frac{1}{\lambda}\operatorname{grad}\Psi\operatorname{grad}\lambda+\frac{c\gamma\nu^2}{\lambda}\Psi=0,\qquad(4)$$

where Δ is the two-dimensional Laplacian.

Hence it follows that the solution of Eq. (1) can be written in the form

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